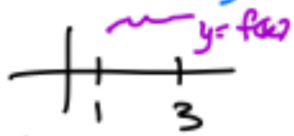
Let $f(x)$ be a twice differentiable function.

$f''(x)$ exists everywhere.

What do these things tell you? (if the statement is true)

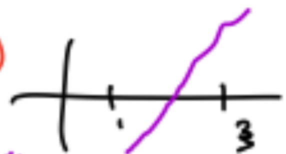
① $f(x) > 0$ for $x \in (1,3)$



② $f(x) < 0$ for $x \in (1,3)$

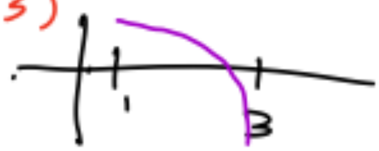
③ $f'(x) > 0$ for $x \in (1,3)$

f is strictly increasing on $(1,3)$ (tangent lines have positive slope)



④ $f'(x) < 0$ for $x \in (1,3)$

f is strictly decreasing on $(1,3)$

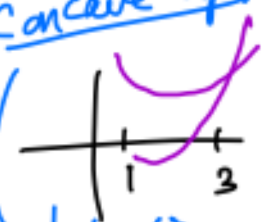


⑤ $f''(x) > 0$ for $x \in (1,3)$

slope f' of f is strictly increasing on $(1,3)$

or

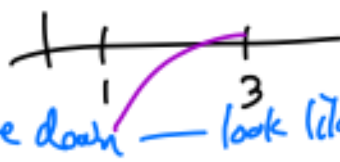
concave up



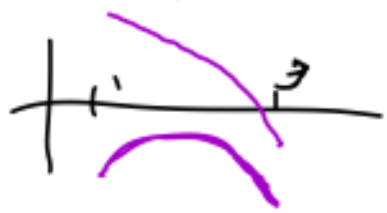
looks like a cup

⑥ $f''(x) < 0$ for $x \in (1,3)$

slope f' of f is strictly decreasing on $(1,3)$



or



Concave down — look like a frown

If x_0 is a critical point of $g(x)$, then $y = g(x_0)$ is the critical value.

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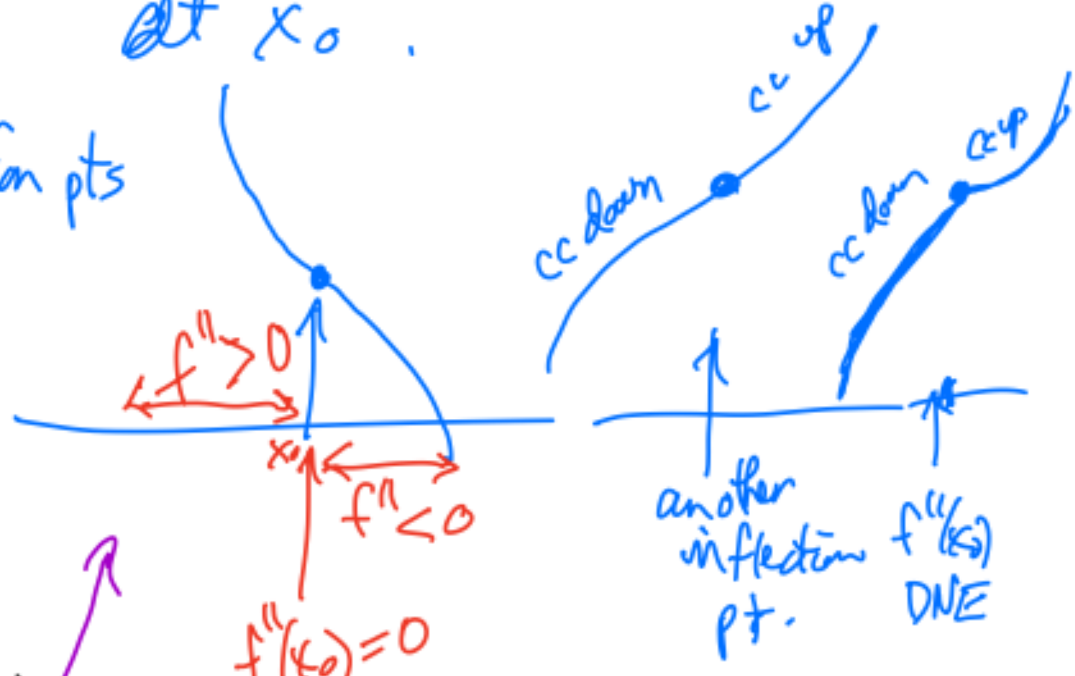
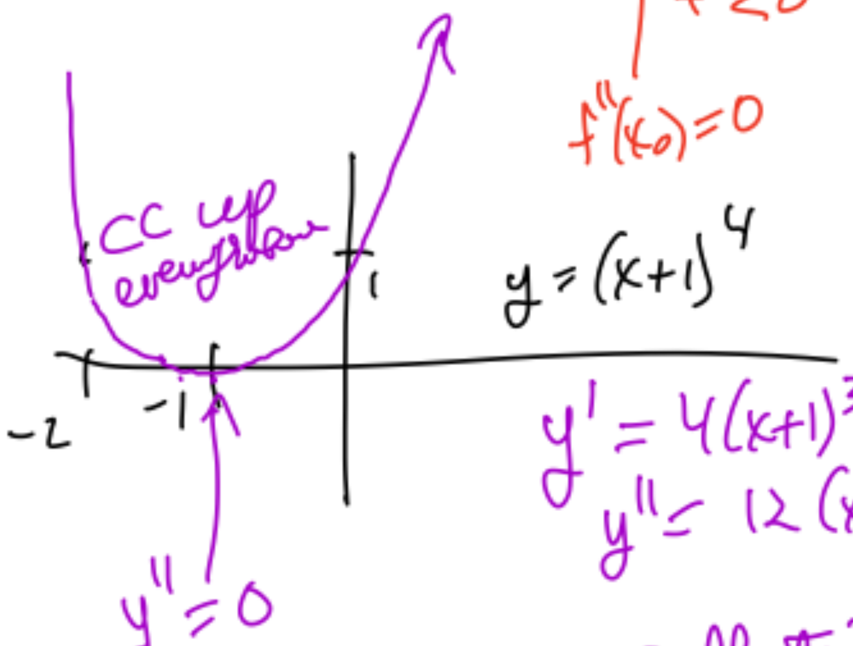
x_0 is an inflection point of a function f iff the function changes from cc up (concave) to cc down or from cc down to cc up at x_0.

inflection pts





$y' = 4(x+1)^3$

$y'' = 12(x+1)^2 = 0$ at $x=-1$.

but $x=-1$ is not an inflection point.

(but it is a critical point).

Example Let $f(x) = e^{-x^2}$.

Find all critical pts & inflections, intervals of increase & decrease.

Solution: $f'(x) = -2xe^{-x^2}$

$$f''(x) = -2e^{-x^2} + 4x^2e^{-x^2}$$

$$= (-2+4x^2)\,e^{-x^2}$$

sagecell.sagemath.org

Type some Sage code below and press Evaluate.

```
f(x)=e^(-x^2)
f1(x)=diff(f(x),x)
f2(x) =diff(f(x),x,x)
f2b(x)=diff(f(x),x,2)
f25b(x)=diff(f(x),x,25)
show(f(x))
show(f1(x))
show(f2(x))
show(f25b(x))

```

Evaluate

$e^{(-x^2)}$

$-2\,xe^{(-x^2)}$

$4\,x^2e^{(-x^2)} - 2\,e^{(-x^2)}$

$-33554432\,x^{25}e^{(-x^2)} + 5033164800\,x^{23}e^{(-x^2)} - 318347673600\,x^{21}e^{(-x^2)} + 11142$